Calculus 30 Exploration Critical Numbers and Maximum/Minimum Values

Tracey Uhrich, RCHS, Grade 12, Calculus

Artifact Summary- Students are grouped together to explore the relationship between the vertex of quadratic functions that they have learned in previous math courses with derivatives and critical numbers in Calculus. By allowing the exploration they are able to think critically, work collaboratively, and use digital technology (graphing calculators or Desmos software) to create connections. At the end of the exploration each group has the chance to speak with me where I invite more conversations about how they made the connection and can apply that to what we are learning in Calculus. I have attached three student work samples to show what they came up with.

A. In your groups discuss the ways you can find the maximum or minimum values for the following functions. Use those methods to actually find the maximum or minimum values.

$$y = x^2 - 4x - 12$$

$$y = -x^2 - 1x + 6$$

$$y = 2x^2 - 7x - 15$$

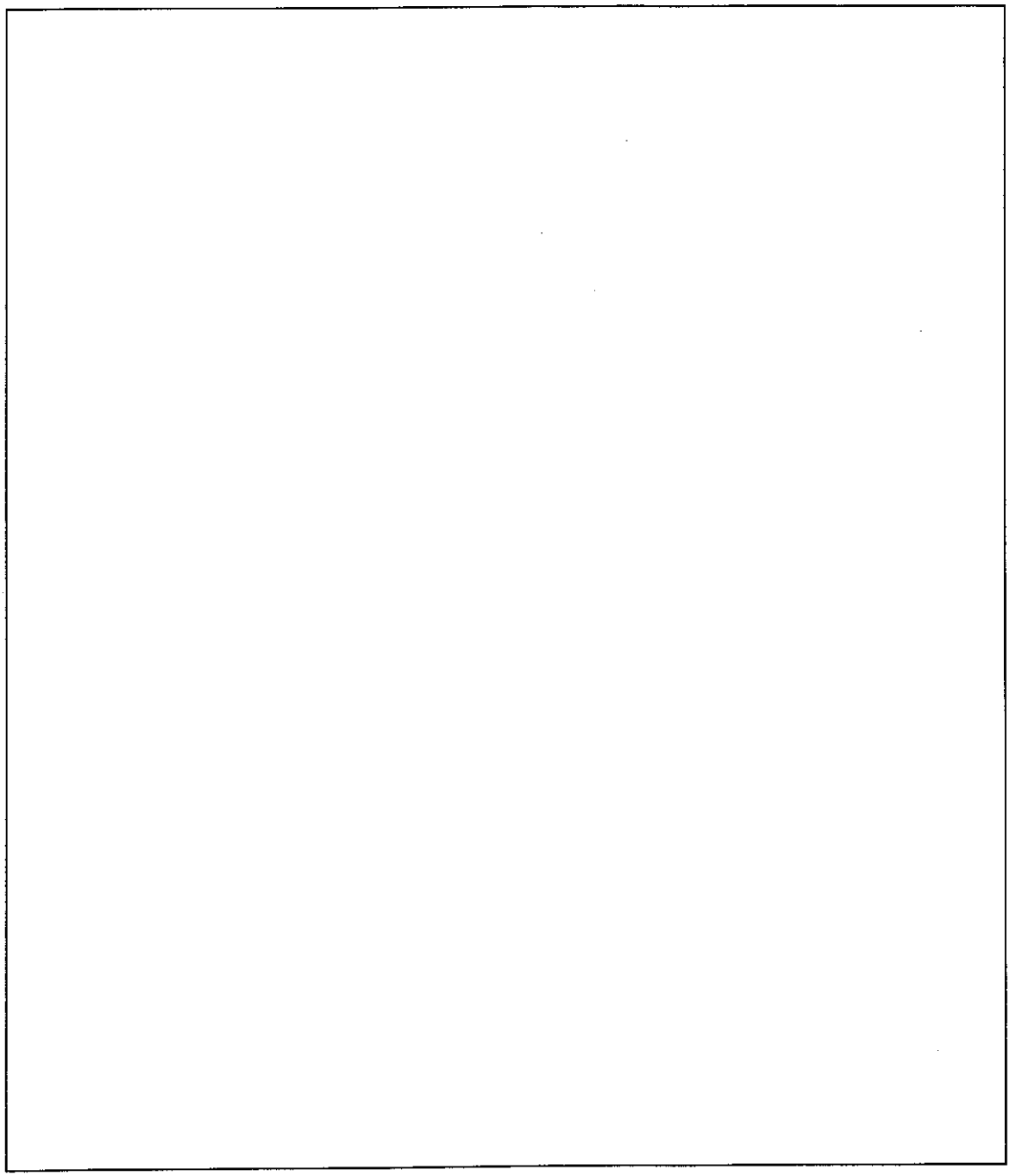
$$y = -x^2 - 1x + 12$$

$$y = 2x^2 - 11x + 12$$

Possible sample questions to guide your discussion:

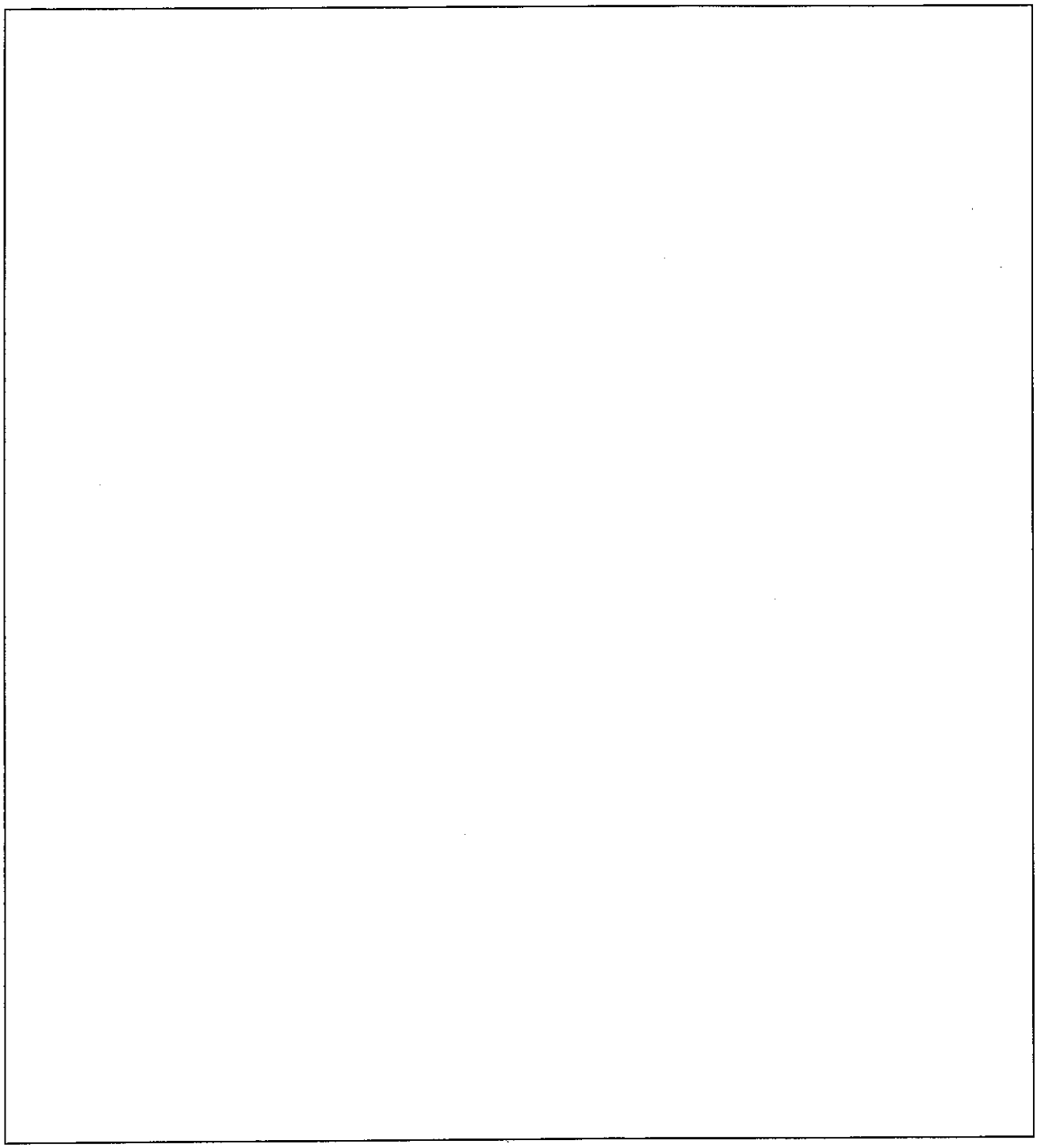
1. What does maximum/ minimum refer to?
2. How can you find the maximum or minimum?
3. What methods can you use to determine the maximum or minimum?

Record your work/thoughts in the space provided:



B. As a group, find the derivative of each of the above functions you found the maximum or minimum for in part A. Set the derivative equal to zero and solve for the variable. What connections can you make to the graphs you have?

Do you work and record connections in this box:



C. Interview time! Your group and I will discuss your findings!

Reflection: This activity went quickly because the student's knowledge of maximum and minimum values was excellent from other math classes. Next time I would add a part D to the activity in which I had the students look at relative maximums and minimums and cubic functions.

① Student Sample

Calculus 30 Exploration Critical Numbers and Maximum/Minimum Values

A. In your groups discuss the ways you can find the maximum or minimum values for the following functions:

$$y = x^2 - 4x - 12$$

$$y = -x^2 - 1x + 6$$

$$y = 2x^2 - 7x - 15$$

$$y = -x^2 - 1x + 12$$

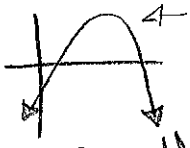
$$y = 2x^2 - 11x + 12$$

you could graph the equations

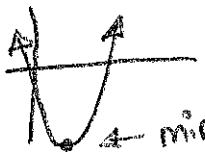
Possible sample questions to guide your discussion:

1. What does maximum/ minimum refer to?
2. How can you find the maximum or minimum?
3. What methods can you use to determine the maximum or minimum?

1. Maximum is the highest point on the graph.
ex)



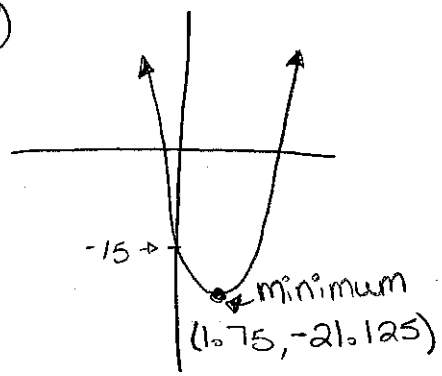
Minimum is the lowest point on the graph.
ex)



2. you could use a graphing calculator, table of values and sign analysis.
3. Same as # 2.

Record your work/thoughts in the space provided:

3.



B. Take the derivative of each of the above functions you found the maximum or minimum for in part A. Set the derivative equal to zero and solve for the variable. What connections can you make?

Do you work and record connections in this box:

(2.) $y = 2x^2 - 7x - 15$

$y' = 4x - 7$

$0 = 4x - 7$

$\frac{4x}{4} = \frac{7}{4}$

$x = 1.75$

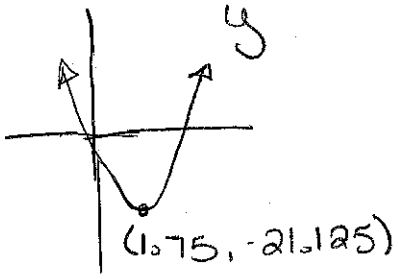
$y = 2(1.75)^2 - 7(1.75) - 15$

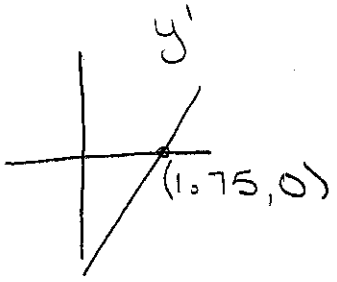
$y = 6.125 - 12.25 - 15$

$y = -21.125$

$(1.75, -21.125)$

You get the same point as before when you graphed the original equation. The 'x' value of the of the derivative of the intercept is the same as the 'x' value of the min.





(2) Student Sample

Calculus 30 Exploration Critical Numbers and Maximum/Minimum Values

A. In your groups discuss the ways you can find the maximum or minimum values for the following functions:

Graph these to find the max

$y = x^2 - 4x - 12$ [1] →

$(\frac{1}{2}, 6.25)$ ← [2] $y = -x^2 - 1x + 6$

$y = 2x^2 - 7x - 15$ [3] →

← [4] $y = -x^2 - 1x + 12$

$y = 2x^2 - 11x + 12$ [5] → (2.75, -3.125) or min

Possible sample questions to guide your discussion:

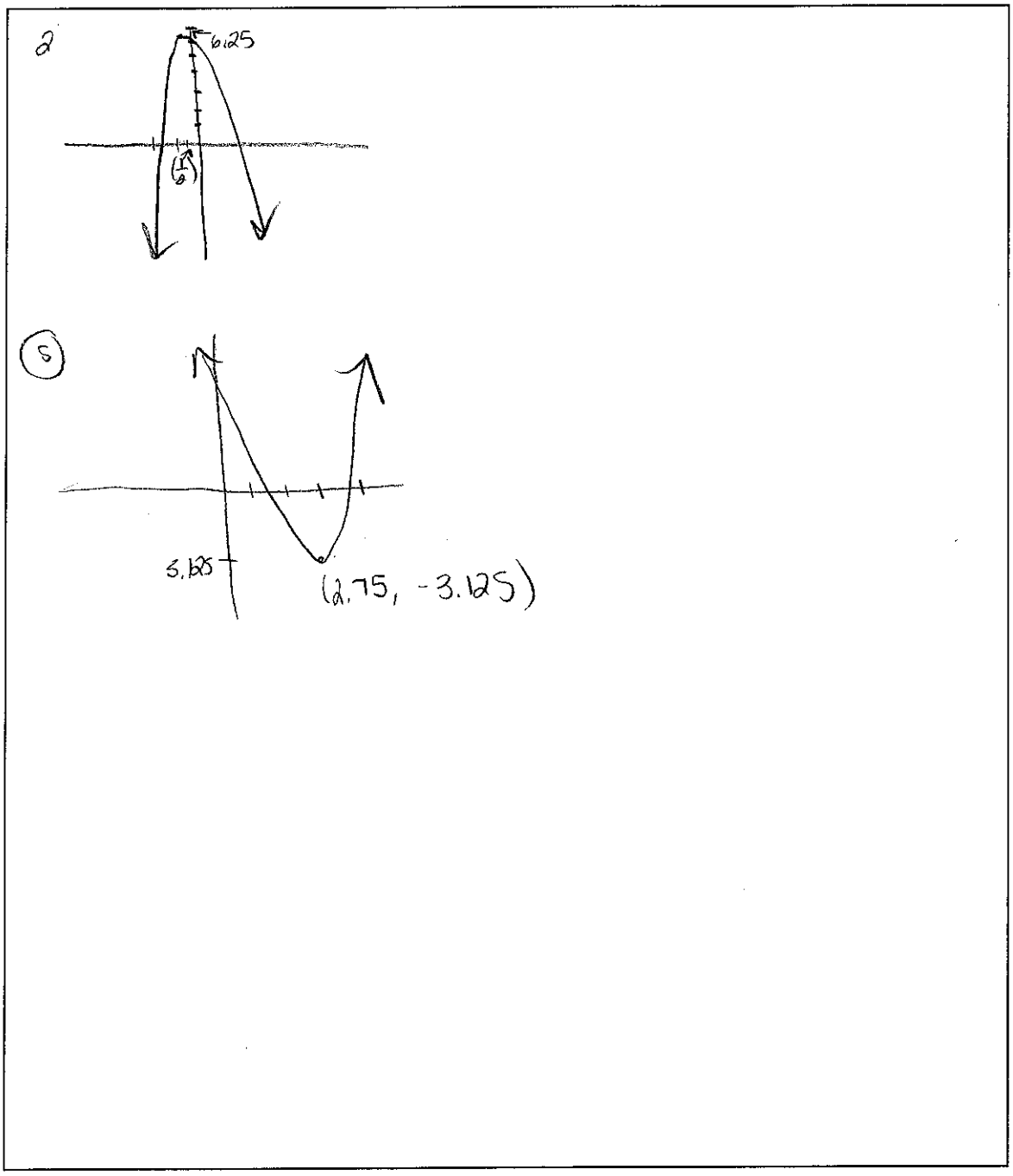
1. What does maximum/ minimum refer to?
2. How can you find the maximum or minimum?
3. What methods can you use to determine the maximum or minimum?

1) The top or bottom point on the graph, Or. The vertex of a parabola

2) Using a table values, sign analysis, or graphing calculator

3) Same as 2

Record your work/thoughts in the space provided:



B. Take the derivative of each of the above functions you found the maximum or minimum for in part A. Set the derivative equal to zero and solve for the variable. What connections can you make?

Do you work and record connections in this box:

$y' = -2x - 1$

$0 = -2x - 1$

$\frac{1}{-2} = \frac{-2x}{-2}$

$-\frac{1}{2} = x$

$y = -(\frac{1}{2})^2 - 1(-\frac{1}{2}) + 6$

$y = 6.25$

$(-\frac{1}{2}, 6.25)$

$y' = 4x - 11$

$11 = 4x$

$x = 2.75$

$8(2.75)^2 - 11(2.75) + 12$

$y = -3.125$

$(2.75, -3.125)$

Connections

- The graph of the derivative is not the same as the original.
- The x intercept of the derivative can be used to find the x value of the max or min
- The derivative gave up the same max/min value as graphing

(3) Student Sample

Calculus 30 Exploration Critical Numbers and Maximum/Minimum Values

A. In your groups discuss the ways you can find the maximum or minimum values for the following functions:

① $y = x^2 - 4x - 12$

② $y = -x^2 - 1x + 6$

③ $y = 2x^2 - 7x - 15$

④ $y = -x^2 - 1x + 12$

⑤ $y = 2x^2 - 11x + 12$

Graphing those!

Possible sample questions to guide your discussion:

1. What does maximum/ minimum refer to?
2. How can you find the maximum or minimum?
3. What methods can you use to determine the maximum or minimum?

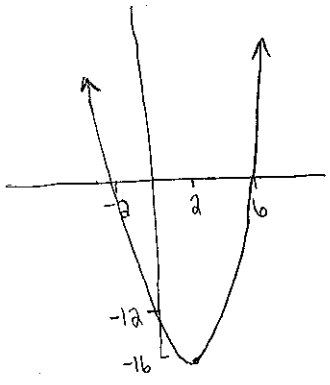
1) max. is the highest point on a graph and min. is the lowest point.

2) you can use your graphing calculator, use a table of values, do a sign analysis.

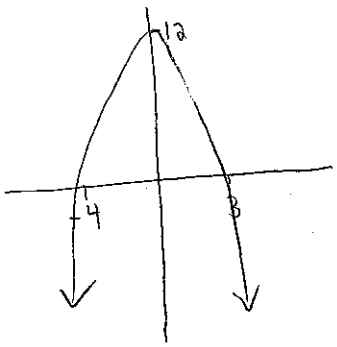
3) The same as #2!

Record your work/thoughts in the space provided:

(1)



(4)



B. Take the derivative of each of the above functions you found the maximum or minimum for in part A. Set the derivative equal to zero and solve for the variable. What connections can you make?

Do you work and record connections in this box:

① $y = x^2 - 4x - 12$

$y' = 2x - 4$

$0 = 2x - 4$

$\frac{4}{2} = \frac{2x}{2}$

$x = 2$

$y = 2^2 - 4(2) - 12$

$= -16$

$(2, -16)$

④ $y = -x^2 - x + 12$

$y' = -2x - 1$

$0 = -2x - 1$

$\frac{2x}{2} = \frac{-1}{2}$

$x = -\frac{1}{2}$

$y = -(-\frac{1}{2})^2 - (-\frac{1}{2}) + 12$

$= \frac{51}{4}$

$(-\frac{1}{2}, 51/4)$

Connections:

- Derivative graphs do not appear the same as the original.
- The x ~~intercept~~ value of the derivative is the x value of vertex
- The coordinants are the same as the max/min of graphs. ~~So the vertex is the max or min?~~