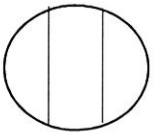
Learning Continuum for Fractions

From *Cognition-Based Assessment and Teaching of Fractions* by Michael T. Battista, 2012.

- It is difficulty for students to move from working with whole numbers to working with fractions:
  - In elementary school, a fraction is a symbolic expression of the form *a/b* where *a* and *b* are whole numbers and b is not zero.
  - In the middle grades, fractions can be positive and negative
  - In high school algebra, fractions can be more complicated expressions
- To understand fractions one must understand a relationship between two quantities – the whole and its parts.
- Critical components to understanding Fractions:
  - Partitioning
  - Iteration
  - Equivalent Fractions, Comparisons, and Operations
  - Fractions on Number Lines
- Jumps in levels are made by students, not teachers, or the curriculum. Teachers can support students by having students work on problems that stretch but do not overwhelm. Have students explain their reasoning and show them alternative ways to solve problems.

Developmental Level	Description
Developmental Level 0	**Student has not concept of the meaning of fractions, but may understand partitioning.** They use whole numbers rather than fractions to describe portions of objects or sets. For example, when asked how much of a rectangle below is shaded, a student at level 0 would say 2 instead of 2/3. 
	Student cannot partition into *equal* parts or subsets.
	Student can partition objects and sets into equal parts, but do not understand these parts as fractions. The student may be able to show how to share in equal pieces, but is not able to name the fraction represented.

Developmental Level 1	**Student Recognize only familiar pictures of fractions.** Student associates fraction names with pictures and may only know fractions such as ½ or ¼.
	Student may not recognize that all parts must be equal. The student may say that 1/3 of this circle is shaded. 
	Student may not be able to conceptualize beyond the denominator. For example: What is ¼ of 12 has to be 4 and not 3.
Developmental Level 2	**Student Understands Fractions as Counting All Parts and Shaded Parts** Student counts all shaded parts; student counts all unshaded parts. They don't have the ideation of the "whole". Students compare fractions by drawing and inspecting pictures. However, the student may not always make the wholes equal. Example: Student may not know that 4/4 and 5/5 are equal.
Developmental Level 3	**Student understands fractions as partitioning a whole shape into equal parts and selecting parts.** The student's reasoning is restricted to operating on whole shapes; the student has difficulties when finding fractions within a set of objects, using improper fractions, and performing arithmetic operations.
	Students use splitting: Splitting is the process of splitting the whole shape into equal pieces. (ex. paper and pencil task)
	Students use iterating: Iterating is the process of putting equal shapes together to make the whole. (ex.pattern blocks)
	The student is able to compare fractions to determine if one is less than, equal to, greater than by drawing and inspecting pictures.

	The student does not fully understand the concept of improper fractions but are beginning to see that an improper fraction is more than one whole.
	The student is at the beginning stages of understanding equivalent fractions but requires the visual/concrete example to determine if fractions are equivalent.
Developmental Level 4	**Student understands Fractions as Partitioning a quantity into equal parts and selecting some parts.** It is at this level that the student develops an understanding of fractions as numbers. The student is able to flexibly go back and forth between the fraction and a picture of the whole or a set of objects.
	The student is able to keep track and maintain the whole and understands improper fractions. The student is able to understand the relationship between an improper fraction and a mixed number.
	The student can use a given picture to establish that two fractions are equivalent. For example: If given a picture of 5/6, the student can show that 10/12 is an equivalent fraction.
4.1	**Student uses pictorial or physical materials to understand and find fractions as quantities.** • Partitioning a numerical quantity • Selecting or accumulating parts Three steps are necessary to make fraction pictures: 1) Represent the whole, 2) Divide the whole into the denominator number of equal parts, 3) Select the numerator number of equal parts. Students should be able to conceptualize the fraction and be able to reverse it. (partition and then iterate)
4.2	**Student uses mental models, not physical or pictorial materials, to understand and find fractions as quantities.** The student has abstracted his previous manipulations of physical and pictorial materials into mental models that enables him to act on numbers rather than visual material.
Developmental Level 5	**Student can manipulate or imagine visual representations of fractions to solve simple fraction arithmetic problems.**

	Comparing Fractions: The student uses pictures to generate families of equivalent fractions to compare by converting them into fractions with common denominators
	Addition and Subtraction: The student can add/subtract with like denominators with mental models. The student would require pictorial or physical representations to add/subtract with unlike denominators.
	Multiplication and Division: Students think in fractions of sets. For example: ¾ X 24 is ¾ of a set of 24. The student begins by partitioning into groups. • Measurement reasoning: how many divisors are in the dividend? • Partitive reasoning: how much is in each part? *Note: Level 5 Reasoning is a prerequisite for making personal and meaningful sense of the symbolic algorithms for fraction arithmetic. Students who perform these algorithms before reaching Level 5 will learn the algorithm by rote.*
Developmental Level 6	**Student uses and has some intuitive understanding of symbolic fraction computation.** • Student is able to symbolically add, subtract, multiply, and divide fractions. • Student is able to determine fractions, compare fractions, and find equivalent fractions. • Student is able to fluently go back and forth between improper and mixed fractions. • Student can use multiplication or division to find fractions of whole numbers. For example: What is ¾ of 144? Divide by 4 and then multiply by 3. • Student can use the Base 10 system to convert fractions to decimals.
Developmental Level 7	Student use pictures or materials to solve difficult fraction arithmetic problems and to understand more precisely why symbolic fraction computations work. • Student is able to understand pictorial justifications of several generalizations concerning mathematical operations on fractions leading to algebraic reasoning. • Student understands the reasoning of algebraic properties.