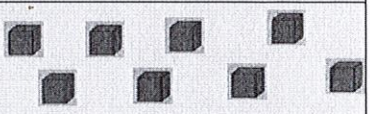
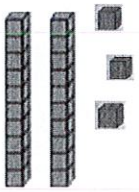
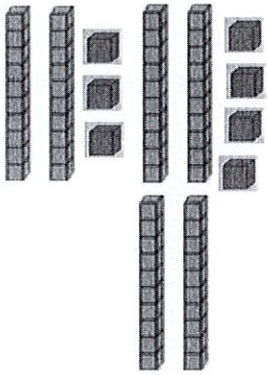
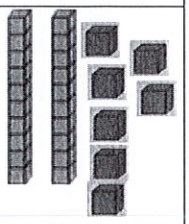
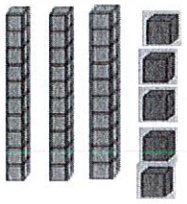
Learning Continuum for Place Value

From *Cognition-Based Assessment and Place Value* by Michael T. Battista, 2012.

- To understand place value students must understand place value to individual numbers and understand pace value in computational algorithms.
- Before students can understand place value they must understand that whole numbers tell us how many objects are in a set. They must understand counting as setting up a one-to-one correspondence between counting word and the objects in a set.
- Counting fingers is more sophisticated than counting objects and counting "counting words" is more abstract than counting fingers. For this reason we should also note if a student needs physical or visible materials to complete the task.
- Jumps in levels are made by students, not by teachers or the curriculum. Teachers can support students by having students work on problems and show them alternative ways of solving problems.
- For students to progress, have them do several problems of a specific type until you see them move to the next level. If they are not ready to move to the next level, try a different kind of problem. If they are still experiencing difficulties try some questions from the previous level. Another strategy is to ask more directive questions or give suggestions that lead the student to work at the next level of reasoning.

Developmental Level	Place Value Description
Developmental Level 0	**Student has difficulties counting by ones.**  Task: How many cubes are there? Response: Student either counts cubes twice or fails to count all cubes.
Developmental Level 1	**Student operates on numbers as collections of ones (no skip-counting by place values).**
1.1	Student correctly counts groups of objects by ones but cannot count groups of ten.  Task: How many cubes are there? Response: 1, 2, 3, 4, ... 20, 21, 22, 23. Student correctly counts based on one-to-one correspondence.
1.2	Student correctly counts groups of tens. They treat groups of ten ones as countable units. However, they continue to count by ones not tens. They are able to count groups of ten when using visible materials.

	Task: There are 38 cookies. How many bags of 10 cookies can be made? Response: Student uses place value blocks and individually counts by ones using the ten blocks. 1, 2, 3 ... 10, 11, 12 ... 20, 21, 22, ... 30. Counts individual blocks 31, 32, ... 38. Counts the ten blocks to get the answer 3.
1.3	Student operates on tens and ones separately as ones. (They do not understand that 1 ten is equal to ten ones. Algorithms are learned by rote and are not understood and not always correctly carried out.) Task: 39 + 46 = Response: 3 + 4 = 7 (writes 7). 9 + 6 = 15 and I don't know if I should write the 1 or the 5. So I am going with the 5 because my mom told me to use the second number (writes 5). So I say 75.
Developmental Level 2	**Student operates on numbers by skip-counting by place value (ex. Count by tens)**
2.1	Student counts by tens and ones separately. Task: Given 4 ten blocks and 7 one blocks, how many square are there altogether? Response: 10, 20, 30, 40, 41, 42, ... 47 Teacher adds two more ten blocks. Response: 10, 20, 30, 40, 50, 60, 61, 62, ...67. Student does not count on from 47. 
2.2	Student counts by tens in mid-decades. Task: In a box, there are 35 red apples and 27 green apples. How many apples are in the box? Response: Use place value blocks to model. 35, 45, 55, (pointing to ones), 60, 62.  

Developmental Level 3	**Student operates on numbers by combining and separating place-value parts (ex. adds tens parts without counting).**
3.1	Student uses multiples-of-ten language ("forty plus twenty equal sixty") Task: In a box, there are 35 red apples and 27 green apples. How many apples are in the box? Response: 30 plus 20 is 50, 5 and 7 is 12. So 50 and 12 is 62.
3.2	Student uses tens language ("4 tens plus 2 tens equals 6 tens") Task: A number has 14 ones and 3 tens. What is the number? Response: 3 tens plus another ten from 14 is 4 tens, plus the 4 ones is 44.
3.3	Student integrates levels 2.1 – 3.2 Task: A number has 23 tens and 6 ones. What is the number? Response: Twenty tens is 2 hundred and 3 tens. So that is two hundred plus 30 which is 230. And 6 ones, that's 236.
Developmental Level 4	**Student understands place value in expanded algorithms.**
4.1	Student understands place value in expanded addition and subtraction algorithms. Task: 342 + 435 Response: *300 + 40 + 2* *400 + 30 + 5* *700 + 70 + 7 = 777*
4.2	Student understands place value in expanded multiplication and division algorithms. Task: 45 X 34 = Response: *30 X 40 = 1200* *30 X 5 = 150* *4 X 40 = 160* *4 X 5 = 20* *1530*

Developmental Level 5	**Student understands place value in traditional algorithms.**
5.1	Student understands place value in traditional addition and subtraction algorithms. Task: 63 – 38 = ? Response: 3 – 8, you can't do that, it would be negative. So I'm going to make this a 50 and put the ten into the 3. Now 13 – 8 = 5. And 50 -30 = 20. So that's 25. *50 13* _ 63 38 25
5.2	Student understands place value in traditional multiplication and division algorithms. Task: 45 X 23 = Response: 3 times 5 is 15. (writes 15), put the ten up here in The tens column. 3 times 4 is 12, plus 1 is 13 (writes 13). 2 times 5 is 10, write the 0 (puts it in the tens column), put The one above the 4. 2 times 4 is 8 plus 1 is 9. Adds the numbers to get 1035. 45 x 23 135 90 1035
Developmental Level 6	**Student generalizes place-value understanding to larger numbers, numbers less than 1, and exponential notation.**